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A Multidimensional Study of the Perceived Structure of Nations

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A MULTIDIMENSIONAL STUDY OF THE PERCEIVED STRUCTURE OF NATIONS

by

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CHAPTER I

THE PROBLEM AND ITS SETTING

The dimensionality of many problems in social psychology, such as in attitudes and values, has been investigated through the use of factor analytic techniques developed for problems in mental testing. A recent extension of psychophysical methods from one dimension to multiple dimensions promises to be of use in social psychology. The present study is a twofold one. Its first aim is to shed some light on the question of the additive transformation required in multidimensional scaling, by means of showing empirically the effect on the factor structure of the choice of various values of the additive constant. Its second aim is to provide a contribution to the psychology of attitude organization, by exploring with regard to a group of students from India a domain defined by a number of nations. It was hoped that the perceptual organization of nations might be uncovered. From this, the variables the students employ in thinking about different nations might be inferred.

Multidimensional scaling, at face value, is the method of choice in a domain of perceptual objects when the important variables cannot be defined in advance. In the present study, the attitudes of the subjects towards different nations might be studied in the more traditional way by requiring them to rate all the nations on several scales. The experimenter would decide on the aspects of nations that are of importance. The correlations of the ratings given to the nations might then be subjected to factor analysis, and the resulting structure would be interpreted as underlying the ratings given to the nations under the various aspects. However, it cannot be assumed that the structure would resemble that resulting from rating the nations under some other set of aspects. Further, it cannot be assumed that any correspondence would exist between the structure so obtained and those aspects of the nations that the subjects actually employ in their thinking about international affairs or in making judgments about nations. A multidimensional scaling approach to this latter question uses the subjects' ratings of the overall similarities and differences among the nations. The structure developed from the ratings is interpretable as the subjects' perceptual organization of the domain. The factors are then interpretable as the variables of the perceptual organization. The advantage of the method is that the experimenter does not specify those aspects of the nations to which the subjects are to attend. Instead, the analysis is carried out in terms of the subjects' own judgment as to which are the variables of importance.

The perceived similarity and perceived difference between two objects is represented as the distance between two points in the multidimensional model. Objects perceived as relatively similar are represented as points relatively close in the multidimensional space. Objects perceived as relatively dissimilar are represented as points relatively far apart.

Young and Householder (1938) have given the conditions under which a set of values may be regarded as the interpoint distances among real points in Euclidean space. They have also given the method for determining their projections on a set of orthogonal axes in the space.

Given a set of n points ($i, j, k, = 1, 2, \ldots n$), and letting d_{ij}, d_{ik}, and d_{jk} be the distances between the points, matrix B_i is an (n-1)(n-1) symmetric matrix with elements

$$b_{jk} = 1/2 \left(d_{ij}^2 + d_{ik}^2 - d_{jk}^2 \right) \qquad 1$$

The element b_{jk} may be considered to be the scalar product of vectors from point i to points j and k. This follows from the cosine law, since

$$d_{jk}^2 = d_{ij}^2 + d_{ik}^2 - 2d_{ij}\, d_{ik} \cos\alpha_{ijk}$$

which rearranged becomes

$$d_{ij}\, d_{ik} \cos \alpha\ jik = 1/2 \left(d_{ij}^2 + d_{ik}^2 - d_{jk}^2 \right) \qquad 2$$

The left hand side of equation 2 is b_{jk} of equation 1. The values b_{jk} form the matrix B_i. If the matrix B_i is positive semi-definite, the distances may be considered to be distances between points lying in a real Euclidean space. The rank of the matrix B_i is equal to the dimensionality of the set of points. The B_i matrix when factored produces a matrix the elements of which are the projections of the points on a set of orthogonal axes, with the point i as origin.

Employing the fact that the average projections of the points on a factor is equal to the projections of the centroid on the factor, Torgerson (1958) presents an equation for the vector products with the centroid as origin. Following Torgerson, the vector product so determined will be designated b^*_{jk} and the matrix of which b^*_{jk} is an element will be designated B^*. The value b^*_{jk} can be determined as follows:

$$b^*_{jk} = 1/2 \left(1/n \sum_j^n d_{jk}^2 + 1/n \sum_k^n d_{jk}^2 - 1/n^2 \sum_j^n \sum_k^n d_{jk}^2 - d_{jk}^2 \right) \qquad 3$$

Torgerson (1951) states that multidimensional scaling may be best considered as involving three basic steps: 1) obtaining the scale of comparative distances; 2) estimating the additive constant for converting comparative distances into absolute distances; 3) determining the dimensionality of the psychological space and the projections of the stimuli on axes of the space from the absolute distances between the stimuli.

The scale of comparative distances may be obtained by several methods, each of which requires the subjects to examine the stimuli in pairs and to judge the similarity of each pair. The scaling methods are essentially adaptations of traditional psychophysical methods, requiring the subject to judge pairs rather than single objects. Among those used have been the method of multidimensional rank order, the method of triads, which is a variant of pair comparisons, and the method of successive intervals. The research here reported employed an adaptation of the method of magnitude estimation (Stevens, 1957) with two values defined in advance by the experimenter in order to fix the scale.

The comparative distances are determined on an interval scale, that is, with respect to an arbitrary origin. The spatial models require that distances be given on a ratio scale, since the dimensionality depends on the absolute value of the distances. The zero point on the scale must be located in order to transform the scale of comparative distance into estimates of absolute distances. This is the additive constant problem.

Torgerson (1958) states that comparative distances are related to the absolute distances by an equation of the form

$$d_{jk} = h_{jk} + c, \qquad 4$$

where d_{jk} is the required absolute distance and h_{jk} is the comparative distance as determined by the experimental data. He suggests that a value for c be used such that it allows the stimuli to be fitted by a real Euclidean space of the smallest possible dimensionality.

In the example presented by Torgerson (1951), five points have the following comparative interpoint distances h_{jk} ($j, k = 1, 2, 3, \ldots 5$, $j \neq k$).

$h_{12} = 1$	$h_{24} = 4$
$h_{13} = 2$	$h_{25} = 0$
$h_{14} = 1$	$h_{34} = 1$
$h_{15} = -1$	$h_{35} = -1$
$h_{23} = 1$	$h_{45} = 0$

With these comparative distances, the value of the additive constant which will allow the stimuli to be fitted by a real Euclidean space of the smallest possible dimensionality is 4. If 4 is added to each of the comparative distances the following absolute distances are obtained:

$d_{12} = 5$	$d_{24} = 8$
$d_{13} = 6$	$d_{25} = 4$
$d_{14} = 5$	$d_{34} = 5$
$d_{15} = 3$	$d_{35} = 3$
$d_{23} = 5$	$d_{45} = 4$

These distances will generate the configuration shown in Fig. 1.

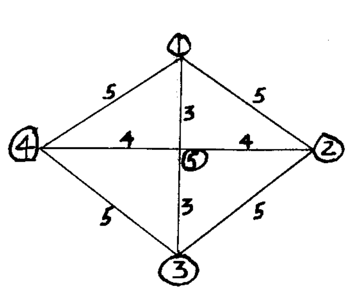

Fig. 1. Configuration generated by fictional distances.

It may be noted that for any smaller value of the additive constant, the points do not exist in a real Euclidean space. For example, if 1, 2, or 3 is added, $d_{45} + d_{25} < d_{24}$, which is an impossible relationship in a real Euclidean space. For any value of the additive constant larger than 4, the points lie in a real space of dimensionality greater than two.

Messick and Abelson (1956) present a general solution for the determination of c. This is based on the theorem that the sum of the roots of a symmetric matrix is equal to the sum of the diagonals. Since B* is symmetric, then,

$$\sum_{j=1}^{n} b^*_{jj} = \sum_{m=1}^{r} \beta_m \qquad 5$$

where β_m is the m^{th} root of matrix B* and r is the number of roots. In their solution, the estimate of c which is desired is the one that gives a relatively small number of large latent roots with the remaining roots small and distributed about zero. A detailed rationale and exposition is to be found in their publication (1956).

The determination of the roots and the projections of the points of the latent vectors is the same procedure, mathematically, as the principal axis method of factor analysis (Thurstone, 1947).

Messick and Abelson (1956) present an example of the effect of c on the final configuration of points. The example employs theoretical values, however, and the guarded conclusion reached is that an underestimate of c will lead to a relatively greater distortion of the final structure than will an overestimate. No estimate of the range of values of c that might safely be employed is given. More to the point, the nature of the effect of c on the final structure has not been deduced, although it has been established that the number of real factors decreases as c decreases.

The factor analysis of a set of variables involves the determination of the projections of vectors representing the variables upon a limited number of axes. The existence of complex dimensions, sometimes termed imaginary dimensions, in that the $\sqrt{-1}$ cannot be represented in the real geometric model postulated for factor analysis, is noted by the presence of negative roots in the matrix. Briefly, a matrix of vector products can be reduced to an equation for the ellipsoid of vectors in space; the length of each of the axes of the ellipsoid is a solution for the equation of the ellipsoid. If the matrix, and therefore the corresponding equation, termed the characteristic equation, has negative roots, it follows that the ellipsoid has negative lengths which are not regarded as meaningful in factor analysis. Strict factor analysis theory does not permit the factoring of such a matrix, unless it may be assumed that the negative roots are small and therefore may be regarded as the resultant of error.

In the factor analysis of tests, the size and number of negative roots depend on the values employed as communalities. The certain avoidance of large negative roots requires the use of unity in the diagonals of the matrix to be factored. The number and the size of negative roots may be controlled by the selection of communalities. A correlation matrix will, in general, have some negative roots for any estimate of the communalities (Harman, 1960, p. 187); their number and size are in general, inversely related to the communalities employed. Iterative factorings will not eliminate or reduce the negative roots, since any iteration procedure involves an arbitrary decision regarding the number of factors to be taken from the matrix.

The factor analysts of intelligence tests or ability tests have ignored the problem in practice. This may be ascribed to the fact that there are no guide lines to follow; if the matrix contains large negative roots, they may be eliminated by an upward adjustment of communalities. But the arbitrary decisions involved in deciding which communalities to adjust, or the size of adjustments required, undoubtedly persuade the factor analyst to ignore the problem. The error component involved in the roots at present cannot be determined; if this could be known, a beginning might be made to this problem in the factor analysis of tests. It is possible that the neglect of this question in practice is one of the reasons for the dearth of replicable results in the factor analysis of tests.

The number and size of negative roots in the correlation matrix depends in a complex way on the communalities estimated for the variables. In the multidimensional matrix, the number and size of negative roots has a more straightforward solution. It is known that relatively small value of c will increase the number and size of the negative roots, while relatively large values of c will decrease their number and size. The question of interest to the research worker, however, is not how well, the formal mathematical requirements are satisfied but whether or not the first several large factors are disturbed by the possible existence of large negative roots. If the first several large factors are approximately identical for the various values of c, and, therefore, for a various number and total size of the negative roots, the problem need receive no more alteration than it has in the factor analysis of tests.

Since the complete solution for c requires an estimate of the sum of the large roots, and even then entails a forbidding amount of work on a desk calculator for a problem involving any sizable number of variables, it is imperative that the systematic effect of c on the final structure be known before the method will have any general research usefulness. If the effect is great, then the complete solution must be determined for any problem, and the method will be limited to use with high speed electronic equipment. If, on the other hand, the final structure shows little change over a considerable range of values for c, and therefore, over a considerable range in the number and size of the negative roots, then the method can be expected to have increasing application. Any solution to the problem in the multidimensional scaling context would probably be of use in the determination of a solution to the corresponding problem in the factor analysis of mental tests.

Therefore, the primary purpose of the study was to determine the nature of the effect of c on the final structure, employing real, experimental data, and to estimate the size of the effect for a range of values of c. Of course, this problem required the use of electronic equipment. If the effect of c can be understood somewhat further, subsequent studies may perhaps be possible with less refined equipment.

CHAPTER II

REVIEW OF THE LITERATURE

Prior to the application of multidimensional scaling to the field of social attitudes, there have been attempts to investigate their multidimensional structures. Several studies can be cited which have utilized the multidimensional approach of factor analysis.

Thurstone (1934) in a factor analysis of correlations among scales of attitude toward the church, the belief in a personal God, Sunday observance, prohibition, divorce, evolution, birth control, war, patriotism, the Germans, and communism, obtained two factors. The first factor was called "radical-conservatism." This included in the radical direction attitudes favorable to evolutionary doctrine, birth control, easy divorce, and communism, and in the conservative direction, attitudes favorable to the church, prohibition, observance of Sunday, and belief in a personal God. It was found that intelligence positively correlated with the radical attitudes. The second factor which included attitudes favorable to war and patriotism in one direction and attitudes favorable to communism and the Germans in the other, was called "nationalism-anti-nationalism."

In another similar study substantiating a "radical-conservatism" factor, Sanai (1950) factored tetrachoric intercorrelations among 16 items on social attitudes using Burt's bipolar analysis techniques. One general and two bipolar factors were found. The general factor was called a factor of progressivism vs. conservatism. The second factor which is bipolar was named a factor of socialism-atheism vs. "social" progressivism. The third factor, also bipolar distinguished between progressive-political attitudes and agnostic-atheistic religious and this was called a factor of socialism vs. atheism. An alternative method was undertaken using Burt's Group Factor Method. He concluded that there was no serious change in the factor picture.

Kulp and Davidson (1945) applied the Spearman two-factor theory to social attitudes. He found that a general factor accounted for the intercorrelations among five sections of a test on international attitudes. The sub-tests dealt with race, national questions, imperialism, militarism, and international cooperation.

Multidimensional scaling finds its beginning in Young and Householder's (1938) description of how a set of points may be uniquely placed in space, when only interpoint distances between them are given. Their theorems form the basis for determining the dimensions of the smallest possible Euclidean space containing such points and for obtaining the projections of the points on a set of orthogonal axes in the space. This method involves converting a matrix of known interpoint distances into a matrix of scalar products taken from any one of the points and then factoring this matrix to yield another matrix the elements of which are the projections of the points on a set of orthogonal axes. The rank of the matrix of projections is the dimensionality of the space.

Prior to World War II, Klingberg (1941) studied the relations between nations. The first of the three studies in the article used the method of equal appearing intervals in estimating the probability of war between 88 pairs of states within ten years of January 1937. In the second study, he applied the method of triadic combinations to judgments of the relative friendliness or hostility among the Great Powers. The seven Great Powers were presented in all possible combinations of three and the subjects had to decide for each triad which two were most hostile and which two were most friendly. In the third study, Klingberg utilized multidimensional methods in an analysis of judgments of friendliness toward each other of the seven Great Powers in 1939. The data subjected to multidimensional analysis were based on judgments obtained from 241 "students of international affairs in the U.S. and Canada." In order to determine the dimensionality of the space, he utilized a matrix M which is discussed by Young and Householder (1938). M is an (n+1) (n+1) symmetric matrix of squared absolute distances bordered by a row and column of unities. The Young and Householder Theorem II states that the dimensionality of a set of points is two less than the rank of matrix M. Since Thurstone (1935) holds that the rank of a matrix is equal to the highest order of the non-vanishing minors which in this case is 7, Klingberg's M matrices could not have ranks higher than five and consequently, the stimuli could be contained within three dimensions. Klingberg then proceeded to construct a three dimensional model and found that all distances between the seven powers fitted into the three dimensional space very well except for the distance between Japan and Russia. Klingberg suggests that a fourth dimension would be needed in order to correct this ambiguous placing of Japan.

While Young and Householder gave adequate procedures for obtaining projections of points on axes from distances when the data are infallible, a number of difficulties arise when fallible data are employed. With fallible data, each point's observed distance from every other point has an error component. A solution to minimize error in the scalar product matrix, derived by Torgerson (1951) places the origin of the coordinate system at the centroid of the points. Torgerson employed multidimensional scaling in a domain known to be of one dimension and in a domain known to be of two dimensions. In his first experiment, he scaled a set of nine grey stimuli by the method of pair comparisons and by the method of complete triads. After one centroid factor was extracted from the vector product matrix with the origin at the centroid, the residuals were negligible. In his second experiment, a set of nine reds differing in lightness and saturation were scaled by the method of complete triads. Two centroid factors were obtained from the vector product matrix. The two factors were rotated to conform to the Munsell values for brightness and saturation.

Messick (1954) applied the multidimensional method to the study of how people perceive attitudes to be organized. The purpose of his study was first to see whether a set of perceived attitude relationships can be adequately represented in dimensional terms and secondly to see if two groups which probably differ with respect to these attitudes perceive them as being structured in different ways. A multidimensional method of successive intervals based upon the Euclidean model of multidimensional psychophysics was developed in order to handle a larger number of stimuli than the number practically possible in the triadic methods. Before applying his procedure to the domain of interest, he tested it in the psychological color space using eight stimuli of constant hue (red) varying in brightness and saturation. Both the previously validated method of complete triads and that of multidimensional successive intervals were applied to the judgments of color similarity obtained from 42 subjects. The structures obtained from both methods were essentially identical and showed excellent linear fits with the factors of Torgerson, as discussed above, and with Munsell values. On the basis of its validation in an area of known dimensionality, Messick regarded the multidimensional method of successive interval accurate enough in a domain of unknown dimensionality, that of the structure of perceived attitudes.

Seven statements were selected from each of the Thurstone scales of attitude toward war, attitude toward capital punishment, and attitude toward the treatment of criminals. These 21 statements, combined in all possible pairs were set up in booklet form for group presentation. Forty third-year male seminary students and 82 male air force officers candidates acted as subjects of this experiment. The two sets of data were analyzed separately. In each case, the origin was placed at the centroid and the Messick and Abelson solution for the additive constant was employed. The dimensional configurations obtained for both groups by the multidimensional scaling methods of successive intervals were compared. The attitude structures perceived by the two diverse groups were essentially identical, the relationships among statements of the three attitudes being adequately structured in terms of two oblique dimensions - a war dimension and a punishment dimension.

Morton (1959) did a multidimensional study to test the hypothesis that the more similar two people are in terms of traits considered important by the group to which they belong, the more friendly they will be with each other. He used a model and a method to express the relationships among the data in terms of the model. The model pictured his subjects as points in a space with every pair of subjects separated by a "friendship distance." The configuration of subjects formed a "friendship space." He employed a multidimensional rank order method to gather data regarding the "friendship distances" among 15 subjects. Two fraternities provided him with samples. Their behavior friendship distances, twice ascertained by multidimensional methods, were found to be stable since the two sets of data showed a high degree of correspondence. He concluded that multidimensional scaling distance values based on paired comparison judgments are adequate quantitative measures of friendship between people when friendship is defined in behavioral terms. The multidimensional scaling values were found to be meaningful, in that they correlated with an established criterion, wishes for future friendship. The study found that one can account for the degree of friendship among members of a group by the extent of their similarity on traits relevant to the norms, interests, and extraneous group associations of the group.

Devane (1960) did a comparative study of a factor analytic and multidimensional scaling determination of the structure of a set of value statements. In order to compare the two methodologies, an experimental procedure was devised that would permit the data obtained to be treated both as a factor analysis and as a multidimensional scaling problem.

A comparison of the distances between the vector termini in the factor analytic configuration and the distances in the multidimensional configuration was performed for several variants of the scaling values. The hypothesis that the two distance estimates would be linearly related for each statement was borne out for some statements and not for the others. It was discovered that the first factors in the factor analysis and the multidimensional configurations were linearly related and that both first factors were negatively related to the means of the statements. It was also demonstrated that the multidimensional definition that yielded relationships with the factor analysis distances at least as clearly and as regularly as the other definitions could be regarded as unidimensional. It was concluded that the tendency of some statements to yield clear relationships between the two distance measures was due to the proportionality of the two first factors. The fact that both first factors were functions of the means precluded any general conclusion regarding the relationship between the two methodologies.

In summary, the studies cited above have shown the attempts to investigate the multidimensional structure of social attitudes by means of factor analysis. They have also shown the application of multidimensional scaling to psychophysical data. They have been extended to the study of the relations between nations, the structure of perceived attitudes, friendship structures, and the structure of a set of value statements. Multidimensional scaling methods have also been shown to reproduce the Munsell dimensions of saturation and brightness. The present study is designed to shed some light on the additive transformation required in multidimensional scaling and to explore the dimensions which students from India use in viewing other nations.

CHAPTER III

PROCEDURE

Multidimensional scaling requires the subject to judge the interpair differences of a set of objects. In this study, the objects were twelve nations chosen from the different continents. The countries and their designations were:

1.	United States	7.	India
2.	Germany	8.	Argentina
3.	Russia	9.	Mexico
4.	Great Britain	10.	Egypt
5.	Burma	11.	Nigeria
6.	Japan	12.	South Africa

Since n(n - 1)/(2) pairs can be formed from n objects, the subjects were required to judge 66 pairs. The subjects were instructed to judge the overall difference of a pair as it seemed to them and to assign a numerical value to indicate this difference. As a guide in judging the difference, they were shown artificial ratings of pairs not employed in the study. They were then instructed to consider that the pair difference Canada-United States has a value of 40, and that the pair difference Sweden-Vietnam has a value of 60. (See Appendix A) Considerations of an a priori nature indicated that the latter pair would variably be rated as more different than the former. This format fixed two points on the response continuum but avoided the difficulties resulting from the use of a fixed set of categories. The subjects were then presented with the pairs typed on 3 x 5 cards. The order of presentation was systematic and irregular, in that the occurence of a nation in consecutive pairs was avoided. To insure understanding of the directions given, the subjects were tested individually and directions were read orally.

The subjects were 30 students from India currently studying in graduate schools of three midwestern universities. Their ages ranged from 21 to 35, with a mean of 27.33. It was decided to use students from India because they are foreigners and form a homogeneous group. It is planned in time to compare the resulting structure to those characteristic of American and other cultural and national groups.

The means and medians of the ratings given to each pair are presented in Table 1 and Table 2 (pp. 40-41). Magnitude estimates of perceptual stimuli often display increasing positive skewness for increasing scale positions. The appropriate measure of central tendency in this event is either the median or the geometric mean. However, the plot of the arithmetic means and the medians for the pairs (Fig. 2) displays little or no trace of the curvilinearity that would appear if such skewness were present in the data. Therefore the greater stability of the arithmetic mean over the median made it the measure of choice in the present problem. The mean of the difference judgments for a pair of nations is thus taken to be the comparative distance between the two points in a multidimensional space.

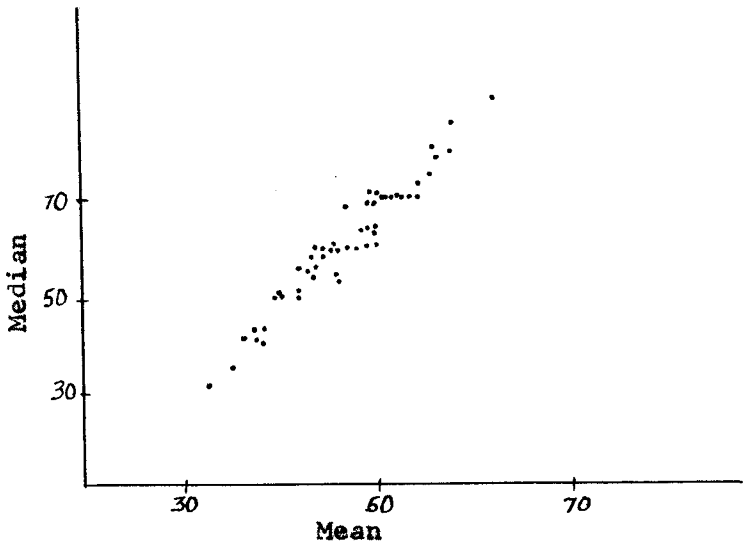

Fig. 2. Plot of the means and medians of differences assigned to the 66 pairs of nations.

The comparative distance estimates were reduced by the constant 42, which reduced the smallest distance to zero.

Messick and Abelson (1956) substitute the expression (h_{jk} + c) for d_{jk} of equation 3, where d_{jk} is the absolute distance, h_{jk} is the comparative distance and c, the additive constant, and present the following expression for the vector product b^*_{jk}, referred to the centroid of all points as origin and with the influence of c separated algebraically:

$$2\, b^*_{jk} = 1/n \sum_{k}^{n} h^2_{jk} + 1/n \sum_{j}^{n} h^2_{jk} - h^2_{jk} - 1/n^2 \sum_{j}^{n}\sum_{k}^{n} h^2_{jk} + 2c\,(1/n \sum_{k}^{n} h_{jk} + 1/n \sum_{j}^{n} h_{jk} - h_{jk} - 1/n^2 \sum_{j}^{n}\sum_{k}^{n} h_{jk}) + c^2\,(\delta_j^k - 1/n) \qquad 6$$

where $\delta_j^k = 0$ when $j \neq k$, and 1 when $j = k$.

The use of equation 6 yields the vector product matrix B^*, with the centroid as origin and with b^*_{jk} as the element employing the comparative distance h_{jk}. Since the value b^*_{jk} depends on the choice of c, the structure resulting from the factor analysis of the matrix B^* might also be expected to vary. One purpose of this study as explained in Chapter I, was to investigate the effect of different choices of c upon the factors determined from B^*.

Four values of c, designed to cover what appeared a reasonable range of values, were employed in the construction of four B^* matrices. These matrices are presented in Tables 3, 4, 5, 6 in the Appendix.

The four B^* matrices were factor analyzed. It is clear that the factor analytic method to be employed in this problem is critical. Any method that does not completely determine the resulting factors would be inappropriate, since it is possible that the factor systems for the various values of c might appear very different, and yet be approximately identical within orthogonal rotations. The appropriate factorial method, therefore, is the principal axis method, since the principal axis solution is completely determined mathematically. Corresponding factors from different solutions may then be compared.

The principal factor analysis and the roots (that is, factor lengths) were determined at the IBM Service Bureau Corporation, New York City. The IBM 704 was employed. The program, from the Applied Mathematics Division of Argonne National Laboratory has the following identification:

AN F202, Eigenvalues and Eigenvectors of a Real Symmetric Matrix

(Fortran II)

Burton S. Garbow - March 31, 1959

Argonne National Laboratory, Lemont, Illinois.

The factors were rotated to find geometrically a simple structure. The factors were rotated without reference to psychological meaning. Eleven rotations were performed.

CHAPTER IV

RESULTS AND DISCUSSION

The effect of the choice of c on the factor structure in multidimensional scaling has been examined with fictitious data (Messick and Abelson, 1956). Mathematical considerations indicate that the number of and size of positive roots will increase with increasing values of c, but it is not clear, either on the basis of the Messick and Abelson example or on mathematical grounds, whether the choice of c will bring about serious alteration in the number of factors or in the factor structure when behavioral data are analyzed and when practically plausible values of c are considered.

Table 7 presents the number and sum of positive roots (which correspond to factor lengths), the number and sum of negative roots (which correspond to complex factors), and the ratios between the two sums for the four choices of c. As expected, (cf. Messick and Abelson, 1956) the number and the size of the positive roots increases with increasing c. What is somewhat surprising is that the number of positive roots undergoes relatively little change for a range of c of apparently considerable extent; the ratio of the positive to the negative sum, increases with c, as expected. Since each positive root corresponds to a real factor, it may be concluded that the number of factors present in the structure is not a serious consideration in the choice of c. Even

Table 7

Number and Sum of Positive and Negative Roots for Four Values of c, and Ratios of Positive to Negative Roots

c	Positive Roots		Negative Roots		Ratio
	Number	Sum	Number	Sum	Pos. sum/ neg. sum
0	8	1947.87	4	-389.31	5.01
2	8	2298.04	4	-382.39	6.01
4	8	2677.40	4	-360.67	7.42
10	10	4075.36	2	-271.77	15.00

the eight factors extracted from the B* matrices when c = 0 or c = 2 are many more than the number of factors of real use with empirical data. The number of interpretable factors from a 12 x 12 matrix would generally be considered to be substantially less than eight. For example, in the factor analysis of mental tests, Thurstone (1947, p.294) points out that a maximum of seven factors can be determined by twelve tests. It is clear, therefore, that an experimenter may safely employ any range of values of c without disturbing the number of interpretable factors.

The relative factor lengths have been shown to be a function of c (Messick and Abelson (1956). For small c, the successive ratios of the first root to the second root, the second to the third, etc., are relatively great; for large c, the successive ratios are relatively small, and approach unity as a limit. In the factor problem, this means that the successive factors, extract a relatively large proportion of the variance when c is small, and the successive factors will be of approximately the same size when c is large. The range of c necessary to show such an effect of practical significance apparently is not mathematically deducible. Table 8 presents the size of successive roots, that is, the lengths of the first five successive factors for the four values of c, expressed as a proportion of the total of the first five roots. It will be noted in Table 8 that the entries within a column are very similar, and that successive factors are of roughly identical lengths, proportionately, for these values of c. Factor length then, is not a serious consideration in the choice of c.

Finally, the critical question in the choice of c is its effect on

Table 8

Proportion Each of First Five Root Is

of Total of First Five Roots

c	Factor I	II	III	IV	V
0	.4862	.2084	.1554	.0916	.0582
2	.4731	.2072	.1602	.0975	.0618
4	.4610	.2065	.1637	.1026	.0658
10	.4315	.2016	.1702	.1146	.0773

factor structure. Since the principal factors are completely determined mathematically, a failure of corresponding factors to be approximately identical within a multiplicative transformation based on the differences in the absolute sizes of the two configurations would indicate that the choice of c is in fact a critical question. Figures 4, 5, 6, 7, 8, and 9 present the plots of corresponding factors for the six comparisons of the four values of c. Only the comparisons of the first four factors are presented; subsequent factors show the same effect. It is clear that the factors did not vary for the range of c presently considered. Factor II appeared to undergo some slight disturbance; the reason for this is not apparent. However, its difference throughout the four configurations is negligible.

In summary, Table 7 indicates that, within the range of c investigated in this study, the choice of c is not critical with regard to the number of factors but that the relative size of the negative roots displays considerable change. Table 8 indicates that the relative factor lengths are only slightly influenced by the choice of c, and Figures 4 through 9 indicate that the factor structure itself, through the first four factors is only negligibly affected. It follows that negative roots of considerable size may be safely tolerated. Mathematical considerations would suggest that the c avoiding or minimizing the negative roots is to be preferred; however, it is clear that the factors of interest were not disturbed by the presence of negative roots. There is some evidence here that the practice of ignoring the problem in the factor analysis of tests is warranted, within limits. It may be concluded that c is not a critical problem in multidimensional scaling, within surprisingly broad limits, and that subsequent investigators may consider themselves free to set the smallest comparative distance well above zero and to proceed without concern for a further additive transformation. The proper number of factors to extract has not yet a rigorous solution, and is largely a matter of judgment in factor analysis in general. The investigator employing multidimensional scaling should perhaps prefer to err on the conservative side, and to stop factoring somewhat sooner than he otherwise would. In this way, a rigorous solution for c might safely be dispensed with.

In general, multidimensional factor systems have been rotated with the intent of matching the factors with the external criteria. (e.g. Morton, 1959) In the present study, the rotations were performed in terms of simple structure criteria. The structure after eleven rotations may be seen in Table 15 and Figure 10. It was concluded that there were no striking indications of further rotations remaining at this point.

It will be remembered that the subjects' task was to judge the differences among the pairs of stimuli. The resulting factors, therefore are dimensions of the judgments of differences. The dimensions of these differences are determined by the stimuli presented. There undoubtedly are important characteristics of nations that could not appear as dimensions in this study since the nations here considered did not provide the continua of differences. For example, if all the stimuli to be judged are large, then size is not a basis for the judgments of differences, and consequently cannot appear as a factor. Further, if one very different stimulus is employed, then the factors.

will be, in general, the various ways in which this stimulus differes from the others. The selection of stimuli is therefore critical, but unfortunately, there are no very useful guide lines to observe in an exploratory study as this at the present. One must necessarily risk the possibility that the stimuli employed do not provide adequate continua for the national aspects attended to by the subjects. That such a difficulty did occur in the present study may be observed in Figure 10, and will be discussed below.

It is clear that the differences between number 3 (Russia) and the rest of the stimuli were striking enough to minimize the differences among the other stimuli. Number 1 (United States) has a similar, though lesser role. The interpretation of factors became then, the problem of considering the different ways in which Russia was seen to differ from the other eleven nations. Such interpretations are necessarily highly tentative. It is clear that subsequent studies of this domain should include nations that will be seen as similar to Russia so that the presence of these on some factors will provide a somewhat firmer basis for interpretation. The meaning of the factors cannot be known therefore, but some comment on possibilities is in order and is presented.

Factor A may be regarded as a cultural-political factor, defined on one pole by Great Britain, the United States, and India, and by Russia on the other. It is assumed that the historical ties between Great Britain and educated Indians would lead the subjects in the study to regard India in this way. The presence of Nigeria on the same pole as Russia does not bolster this interpretation, but neither does it argue against it.

Factor B may be a geographic factor, a statement, which rests mainly on the clustering of the United States, Argentian, and Mexico on one pole, with Russia defining the extreme of the other pole. Nigeria's position is again anomalous.

Factor C permits two interpretations, both highly speculative. It may be an ethnic or racial factor, which implies that the subjects regard Russia as an European nation. It has also been suggested that it is a pacifism-militarism factor. In view of Russia's extreme position on this factor, one may only say with any certainty that it has to do with a difference between Burma and India on one hand and Russia on the other, which admits of a wide spectrum of interpretations. The position of the United States and Germany, nearer to Russia than to Burma and India, limits the interpretation only somewhat.

Factor D seems to be based largely on differences between Russia and Africa. No further statement is warranted. No attempt was made to interpret Factor E.

CHAPTER V

SUMMARY AND CONCLUSIONS

This study had two aims. The first was to investigate the effect of various values of the additive constant required in multidimensional scaling upon the factor structure with experimental data. Prior to this study, the only discussions available considered fictitious data. The second aim was to determine the dimensions that people use in thinking about nations. Eventually a comparison of the dimensions used by subjects of different cultural backgrounds will be attempted. It is planned to compare the structure of nations as seen by American students to that seen by foreign students. The subjects who contributed to the present study were 30 students from India currently studying in the United States.

The names of twelve nations were arranged in all possible pairs. The subjects were instructed to judge the extent of the overall difference between the nations in each pair. A variation of the method of magnitude estimation was employed. The mean of the difference estimates assigned to each pair, reduced by a constant was then considered to be the distance between the corresponding points in the multidimensional space.

Vector product matrices were generated with the origin in each case at the centroid of the points, employing four different values of c, ranging from zero to ten. The complete principal axis solution was determined for each matrix.

The four unrotated principal axis structures were compared. It was found that the number and size of the negative roots (which imply complex dimensions) decreased as c increased, which was to be expected (Messick and Abelson, 1956). It was also found that, for the first four factors, the relative sizes of successive factors were almost identical from one structure to another, and that these factors were linearly related with a very close fit. It was concluded that future experiments employ an additive constant such that the smallest distance is well above zero, and the factor structure obtained will approximate that to be obtained with any plausible value of the additive constant, with only negligible differences. The number of factors will increase with the value of c. In general, the problem of the number of factors to extract has no rigorous solution. Since the factors obtained with any value of c apparently diminish at approximately the same rate, the experimenter using a multidimensional scaling method need only to aim for a sizable number of positive roots, which imply real factors. He may then stop factoring on the basis of any conventional rule of thumb.

It is clear that similar rotations would be in order for any of the four factor structures, and that the configurations resulting from the same rotation would be identical. The structure resulting from a c of ten was selected for rotation; the same results would be obtained from the rotation of one of the other three. It was found that the very large distances between the point representing Russia and the other points resulted in its having a very large projection on each of the first five rotated factors. The observed differences among the nations must therefore be interpreted as aspects of Russia. In future studies of this domain, care should be taken to avoid this artifact by avoiding the use of one nation so drastically different from the others.

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APPENDIX A

INSTRUCTIONS

You will be shown a series of cards. On each card will be typed the names of two countries. For each pair, you are to decide how different the two countries seem to you. Then assign a numerical value to the pair to indicate how different they seem. Judge their overall difference, as it seems to you, and assign a value. If they seem to you to be not very different, assign a relatively low value. If they seem to you very different, assign a relatively high value.

As a guide, use these values:

Canada 40
United States

Sweden
Vietnam 60

Then if a pair of countries seems to be not as different from each other as Canada and the United States, assign some value less than 40, depending on how much the difference seems to be. If, on the other hand, a pair seems even more different than Sweden and Vietnam, assign some value greater than 60, depending on how much greater the difference appears. If the difference is greater than the first pair above, but less than the second, assign a value between 40 and 60.

For example, one student decided that the pair Peru and Ecuador have a very small difference, that the pair Italy and France have a somewhat greater difference, and that the pair Iceland and Syria have a very great difference.

Peru
Ecuador 36

Italy
France 44

Iceland
Syria 66

APPENDIX B

TABLES

Table 1

Mean of Difference Judgments for the Twelve Nations

Nations	1	2	3	4	5	6	7	8	9	10	11	12
1. U. S.												
2. Germany	47											
3. Russia	63	56										
4. Gr. Britain	46	45	61									
5. Burma	67	61	66	61								
6. Japan	50	49	60	51	52							
7. India	65	58	64	54	42	52						
8. Argentina	58	57	66	61	57	57	56					
9. Mexico	56	62	65	59	55	59	57	47				
10. Egypt	65	59	62	60	53	60	48	52	53			
11. Nigeria	73	62	68	65	54	60	59	55	53	50		
12. S. Africa	64	61	68	60	56	62	62	55	54	54	48	

Table 2

Median of Difference Judgments for the Twelve Nations

Nations	Nations 1	2	3	4	5	6	7	8	9	10	11	12
1. U. S.												
2. Germany	45											
3. Russia	60	52										
4. Gt. Britain	46	43	60									
5. Burma	64	60	65	60								
6. Japan	50	50	56	50	50							
7. India	60	56	60	53	41	54						
8. Argentina	55	60	62	50	55	55	55					
9. Mexico	52	60	62	60	55	57	55	46				
10. Egypt	60	60	60	55	55	60	45	50	54			
11. Nigeria	70	60	65	60	54	60	55	55	54	50		
12. S. Africa	60	60	68	57	55	60	60	54	55	52	46	

Table 3

B* Matrix (Vector Product Matrix) Obtained through Use of Equation 6, c = 0

	1	2	3	4	5	6	7	8	9	10	11	12
1	215.06											
2	137.77	85.48										
3	40.94	98.65	307.81									
4	151.77	90.48	25.65	104.48								
5	-149.44	-82.23	-78.56	-72.73	111.06							
6	106.15	48.85	22.52	42.35	36.15	61.23						
7	-110.56	-38.85	-41.69	26.65	101.94	27.02	92.81					
8	20.44	-28.85	-93.19	-87.35	-16.06	-40.98	-10.69	81.81				
9	49.65	-117.15	-70.48	-52.14	11.15	-73.77	-25.98	68.52	80.23			
10	-119.19	-63.98	-8.31	-71.98	32.81	-93.60	66.19	28.69	17.40	75.56		
11	-282.65	-66.94	-93.77	-121.94	73.85	-41.06	- 7.77	46.73	69 .94	96.10	180.65	
12	-59.94	-63.23	-109.56	-35.23	32.06	-94.85	-79.06	30.94	42.65	40.31	146.85	149.06

Table 4

B* Matrix (Vector Product Matrix) Obtained through Use of Equation 6, c = 2

	1	2	3	4	5	6	7	8	9	10	11	12
1	252.42											
2	156.97	110.52										
3	41.80	107.35	360.17									
4	173.97	108.52	25.35	131.52								
5	-169.08	-96.03	-88.70	-85.53	138.42							
6	118.51	57.06	22.38	47.56	39.51	84.59						
7	-127.36	-47.81	-48.99	26.69	126.14	29.22	117.85					
8	18.30	-35.15	-103.83	-100.65	-21.20	-48.12	-14.99	108.17				
9	51.35	-133.60	-79.28	-61.60	9.85	-85.07	-32.44	82.72	106.27			
10	-136.17	-75.12	-11.79	-84.12	34 .83	-107.58	77.05	32.21	18.76	100.24		
11	-310.44	-78.90	-104.07	-138.90	79.06	-49.86	-13.73	49.43	76.48	107.96	215.69	
12	-70.24	-73.69	-120.36	-42.69	32.76	-108.15	-91.52	33.14	46.69	43.67	167.40	183.10

Table 5

B* Matrix (Vector Product Matrix) Obtained through Use of Equation 6, c = 4

	1	2	3	4	5	6	7	8	9	10	11	12
1	293.45											
2	175.84	139.23										
3	42.33	115.72	416.20									
4	195.84	126.23	24.72	162.23								
5	-189.05	-110.16	-99.66	-98.66	169.45							
6	130.54	64.92	21.91	52.42	42.54	111.62						
7	-144.49	-57.10	-56.62	26.40	150.01	31.09	146.56					
8	15.83	-41.78	-114.80	-114.28	-26.67	-55.59	-19.62	138.20				
9	52.72	-150.40	-88.41	-71.40	8.22	-96.70	-39.23	96.59	135.98			
10	-153.48	-86.59	-15.60	-96.59	36.52	-121.89	87.58	35.40	19.79	128.59		
11	-338.56	-91.19	-114.70	-156.19	83.92	-58.99	-20.02	51.80	82.69	119.49	254.40	
12	-80.87	-84.48	-131.49	-50.48	33.13	-121.78	-104.31	35.01	50.40	46.70	187.60	220.81

Table 6

B* Matrix (Vector Product Matrix) Obtained through Use of Equation 6, c = 10

	1	2	3	4	5	6	7	8	9	10	11	12
1	438.54											
2	230.45	247.36										
3	41.91	138.82	606.29									
4	259.45	177.36	20.82	276.36								
5	-250.96	-154.55	-132.58	-140.05	284.54							
6	164.62	86.53	18.50	65.03	49.62	214.70						
7	-197.89	-86.98	-81.51	23.52	219.61	34.70	254.69					
8	6.41	-63.68	-149.71	-157.18	-45.09	-80.00	-35.51	250.29				
9	54.82	-202.77	-117.80	-102.77	1.32	-133.60	-61.60	136.20	247.10			
10	-207.41	-123.00	-29.04	-136.00	39.59	-166.83	117.16	42.96	20.87	235.64		
11	-424.97	-130.06	-148.60	-210.06	96.63	-88.39	-40.90	56.90	99.31	152.08	392.52	
12	-114.76	-118.85	-166.89	-75.85	32.24	-164.68	-144.69	38.61	59.52	53.79	246.23	355.94

Table 9

Principal Factor Solution and Roots for

First Four Roots when c = 0

	Factor I	II	III	IV
1	14.6357	8.6444	.8623	1.7125
2	8.9587	-1.2243	-1.6079	-4.2456
3	9.2768	-10.4126	-10.1352	2.7611
4	8.8866	.8449	3.1889	-3.9631
5	-7.5152	-5.1532	6.2813	-.2309
6	5.6471	-2.2147	5.5415	-1.8562
7	-2.9012	-6.3893	7.8575	2.7780
8	-3.8044	6.1616	-.6494	4.5040
9	-4.3650	6.9564	-1.3245	5.5029
10	-7.1326	-2.8119	-2.3054	3.0653
11	-13.7447	-1.2777	-3.4785	-3.8706
12	-7.9735	6.1046	-4.2308	-6.1588
Roots	893.1567	382.8128	285.6144	168.3892

Table 10

Principal Factor Solution and Roots for

First Four Roots When c = 2

	Factor I	Factor II	Factor III	Factor IV
1	15.5540	9.0949	1.4393	1.9245
2	9.6360	-1.0415	-1.7412	-4.6114
3	9.5620	-10.6193	-11.7746	3.1670
4	9.7524	.8506	3.4078	-4.4104
5	-7.8898	-5.2728	6.7786	-.2484
6	6.2469	-2.6820	5.8242	-2.0463
7	-3.4929	-7.5352	8.2868	2.9097
8	-4.5358	6.7350	-.3711	5.2575
9	-4.6560	7.5994	-1.0487	6.1808
10	-7.7987	-2.9381	-2.5025	3.2185
11	-14.4532	-.8420	-3.9775	-4.4837
12	-8.6449	6.7726	-4.3219	-6.8637
Roots	1021.4950	447.3915	345.9007	210.5826

Table 11

Principal Factor Solution and Roots for

First Four Roots When c = 4

	I	II	III	IV
1	16.4534	9.5137	2.0304	2.1228
2	10.5267	-.8196	-1.8628	-4.9674
3	9.9617	-10.7013	-13.4009	3.5559
4	10.6169	.8824	3.6380	-4.8310
5	-8.3847	-6.1999	7.1745	-.2816
6	6.6905	-3.1982	6.0814	-2.2297
7	-3.7474	-8.6475	8.6198	3.0501
8	-4.8360	7.3193	-.0538	5.9908
9	-4.9376	8.2583	-.7105	6.8529
10	-8.3910	-3.0705	-2.7204	3.3695
11	-15.0976	-.3756	-4.4157	-5.0770
12	-9.2464	7.4608	-4.3662	-7.5649
Roots	1154.2030	516.9298	409.6682	256.7283

Table 12

Principal Factor Solution and Roots for

First Five Roots When c = 10

	Factor I	II	III	IV	V
1	19.0449	10.6495	3.7440	2.6684	.2864
2	12.7795	-.6845	-2.1561	-6.0023	-.8424
3	11.2156	-10.7401	-18.0583	4.6689	2.3861
4	13.1183	.7997	4.3554	-6.0166	-7.3033
5	-9.5684	-9.2186	8.0286	-.4170	4.6589
6	7.9867	-4.3530	6.7574	-2.7502	9.1383
7	-4.5930	-11.9671	9.3109	3.4985	-4.9528
8	-5.4190	8.8609	.9703	8.1157	1.7124
9	-6.0608	9.8298	.3036	8.8372	1.5188
10	-20.0201	-3.2438	-3.4082	3.8232	-8.6862
11	-17.1993	.9111	-5.4696	-6.7893	3.3132
12	-10.7249	9.4966	-4.3792	-9.6550	-1.2306
Roots	1577.6800	753.6869	622.2963	419.0160	282.7484

Table 13

Final Transformation Matrix

	A	B	C	D	E
I	.0819	.0545	-.3470	.4049	-.0380
II	.3203	-.6529	-.1135	-.4163	-.0676
III	.7497	-.5806	.9256	.0444	-.5930
IV	-.1935	-.4817	-.0955	.8129	.2349
V	-.5396	.0402	-.0285	.0000	-.7662

Table 14

Cosines of Reference Vectors

	A	B	C	D	E
A	1.0000				
B	-.5684	1.0000			
C	.6830	-.4606	1.0000		
D	-.2242	-.1235	-.1298	1.0000	
E	-.1013	.2424	.5286	.1774	1.0000

Table 15

Loadings on the Final Rotated Factors

	A	B	C	D	E
1	7.107	-9.363	-4.615	5.611	-3.256
2	.827	5.253	-5.755	.438	.075
3	-18.251	15.955	-19.901	12.005	10.277
4	9.701	.269	.345	.280	1.047
5	-.151	1.224	11.705	-.018	-7.442
6	-.073	1.046	3.980	2.843	-11.664
7	4.767	.273	11.377	6.380	.079
8	.627	-10.484	.949	.758	-.374
9	.350	-11.120	.381	.652	.298
10	-.467	1.360	.573	.251	10.174
11	-5.691	5.047	1.356	-13.103	-.298
12	1.413	.359	-.453	-16.216	1.037

APPENDIX C

GRAPHS

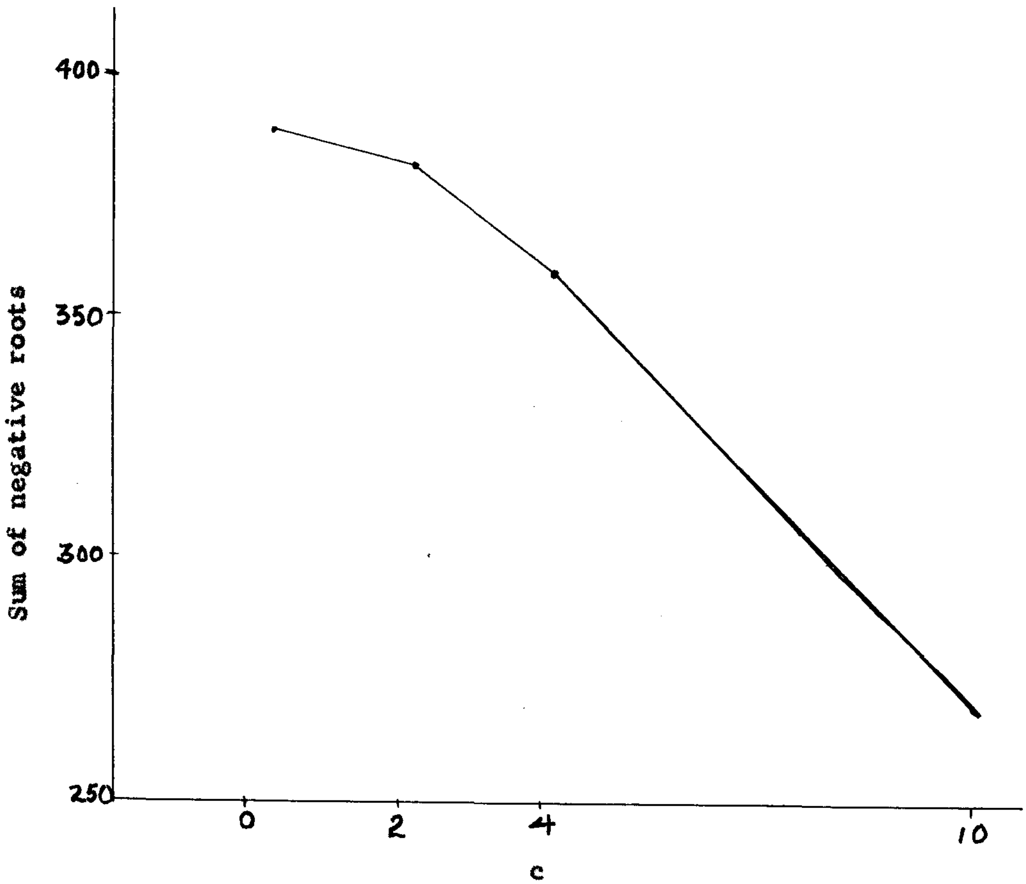


Fig. 3. Plot of the sum of the negative roots for the four values of c.

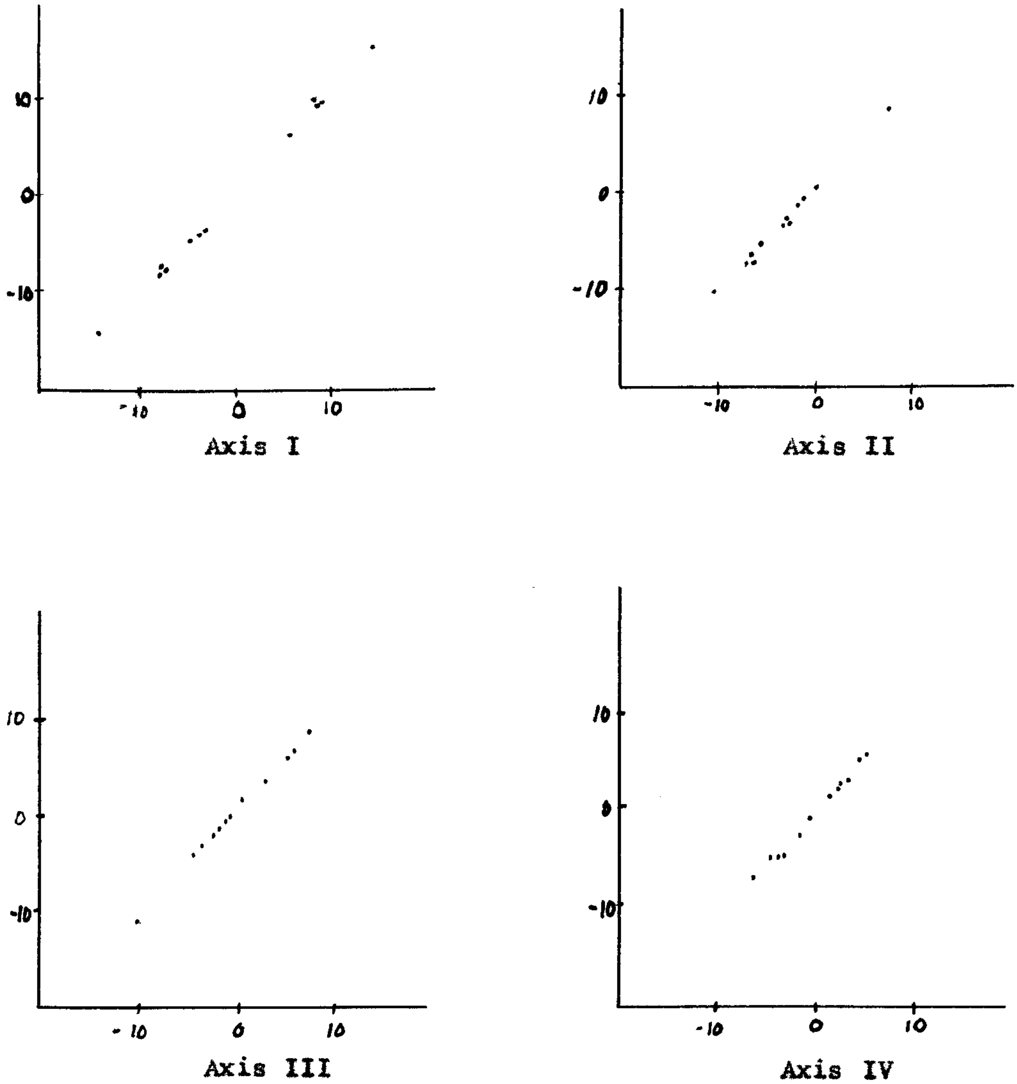


Fig. 4. Relationships between loadings on corresponding principal axes for c = 0 (abscissa) and c = 2 (ordinate) for four largest principal axes.

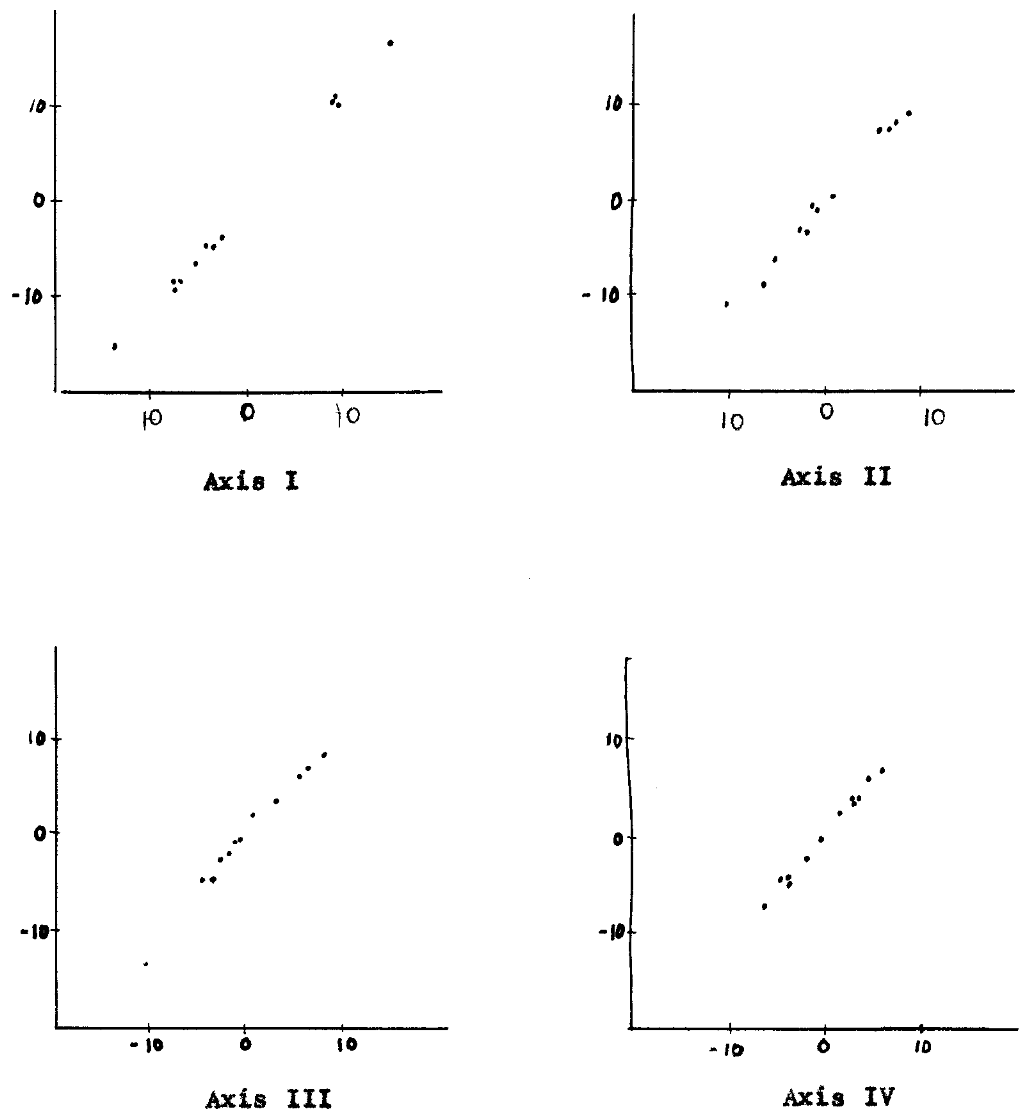


Fig. 5. Relationships between loadings on corresponding principal axes for c = 0 (abscissa) and c = 4 (ordinate) for four largest principal axes.

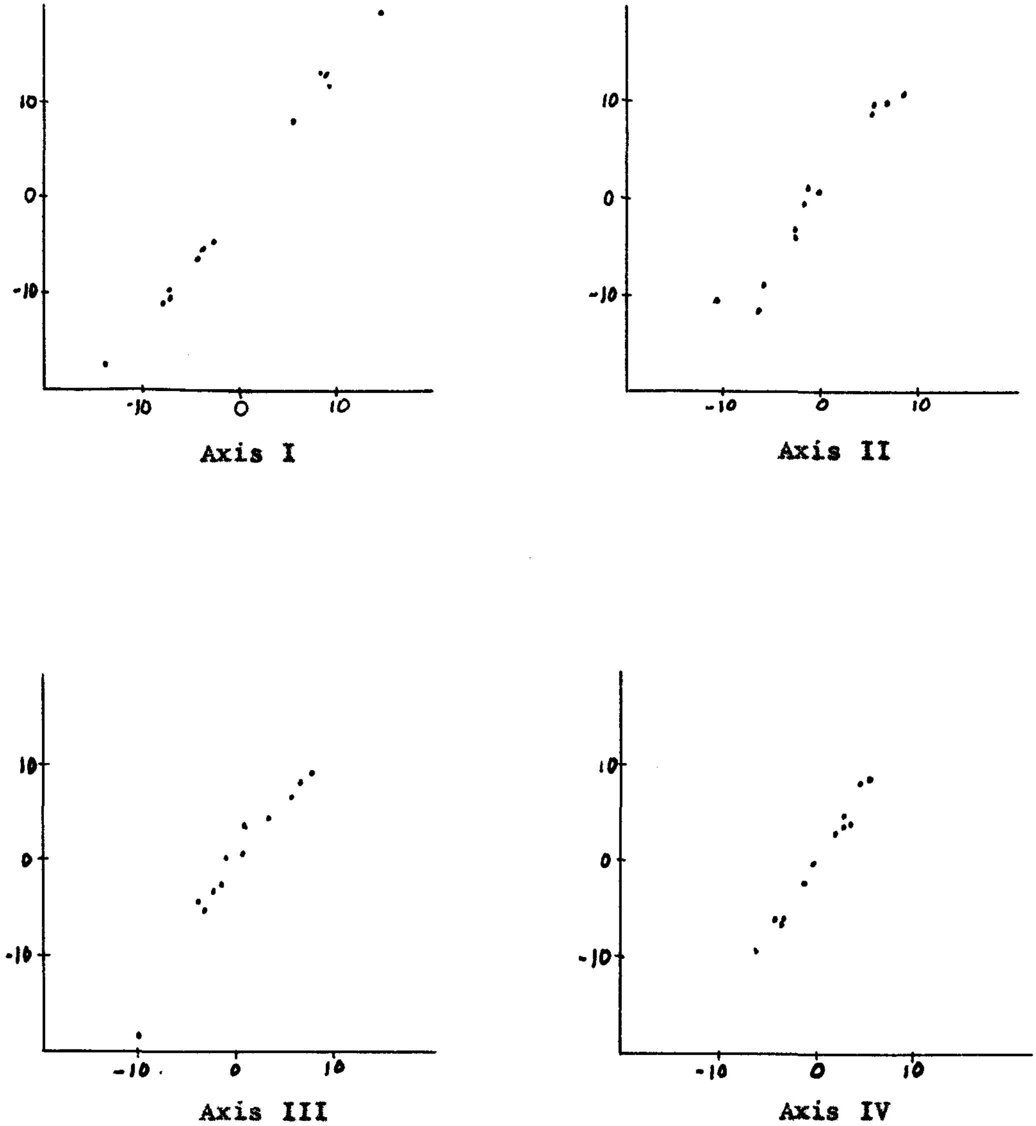


Fig. 6. Relationships between loadings on corresponding principal axes for c = 0 (abscissa) and c = 10 (ordinate) for four largest principal axes.

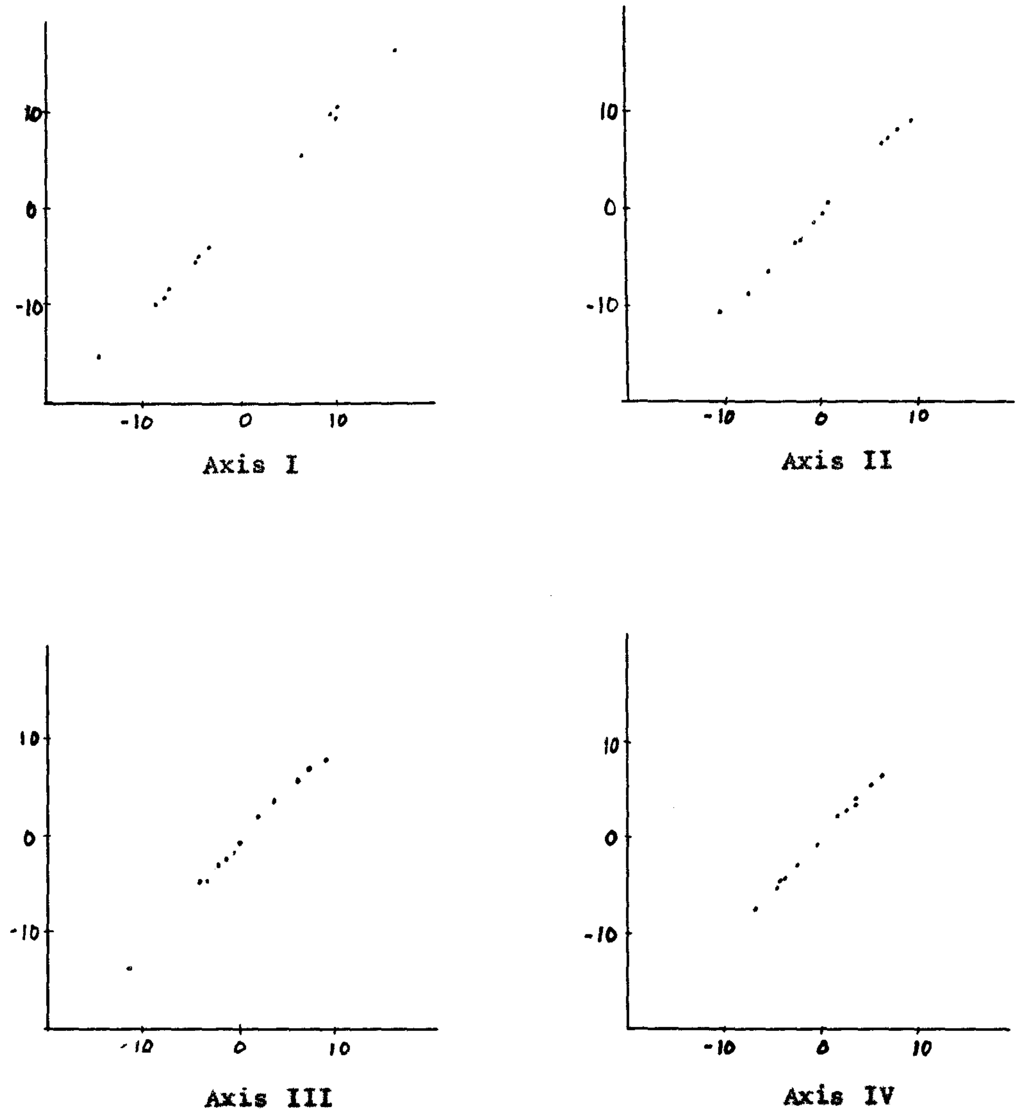


Fig. 7. Relationships between loadings on corresponding principal axes for c = 2 (abscissa) and c = 4 (ordinate) for four largest principal axes.

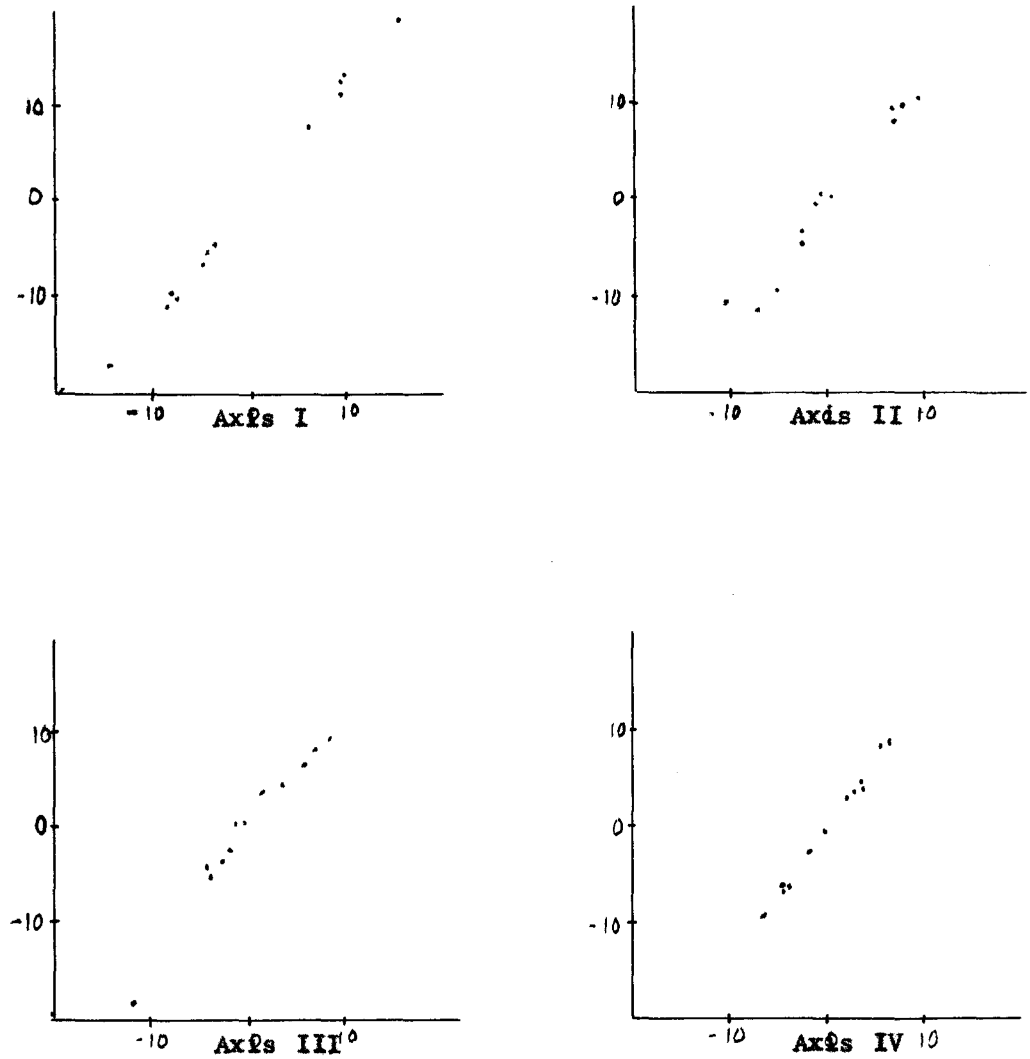


Fig. 8. Relationships between loadings on corresponding principal axes for c = 2 (abscissa) and c = 10 (ordinate) for four largest principal axes.

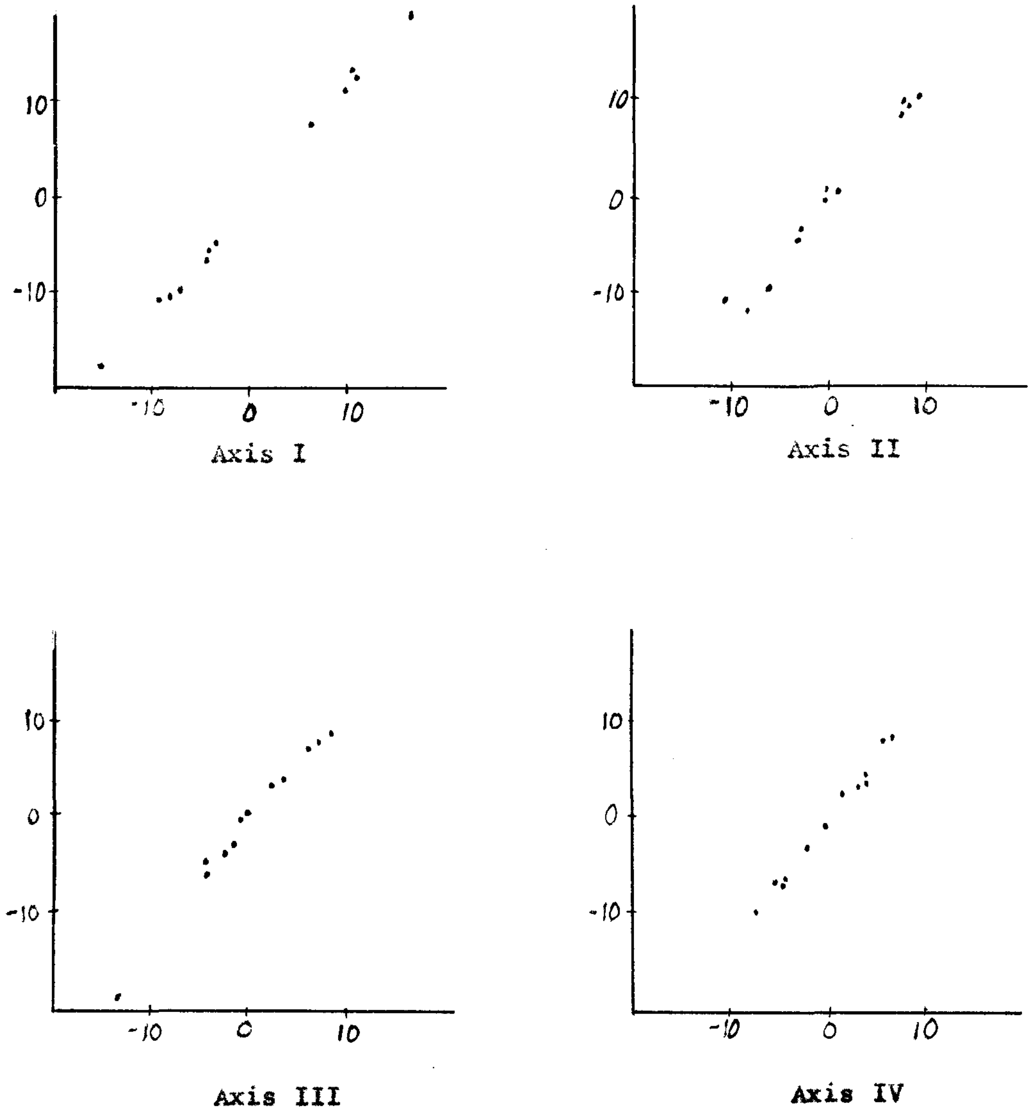


Fig. 9. Relationships between loadings on corresponding principal axes for c = 4 (abscissa) and c = 10 (ordinate) for four largest principal axes.

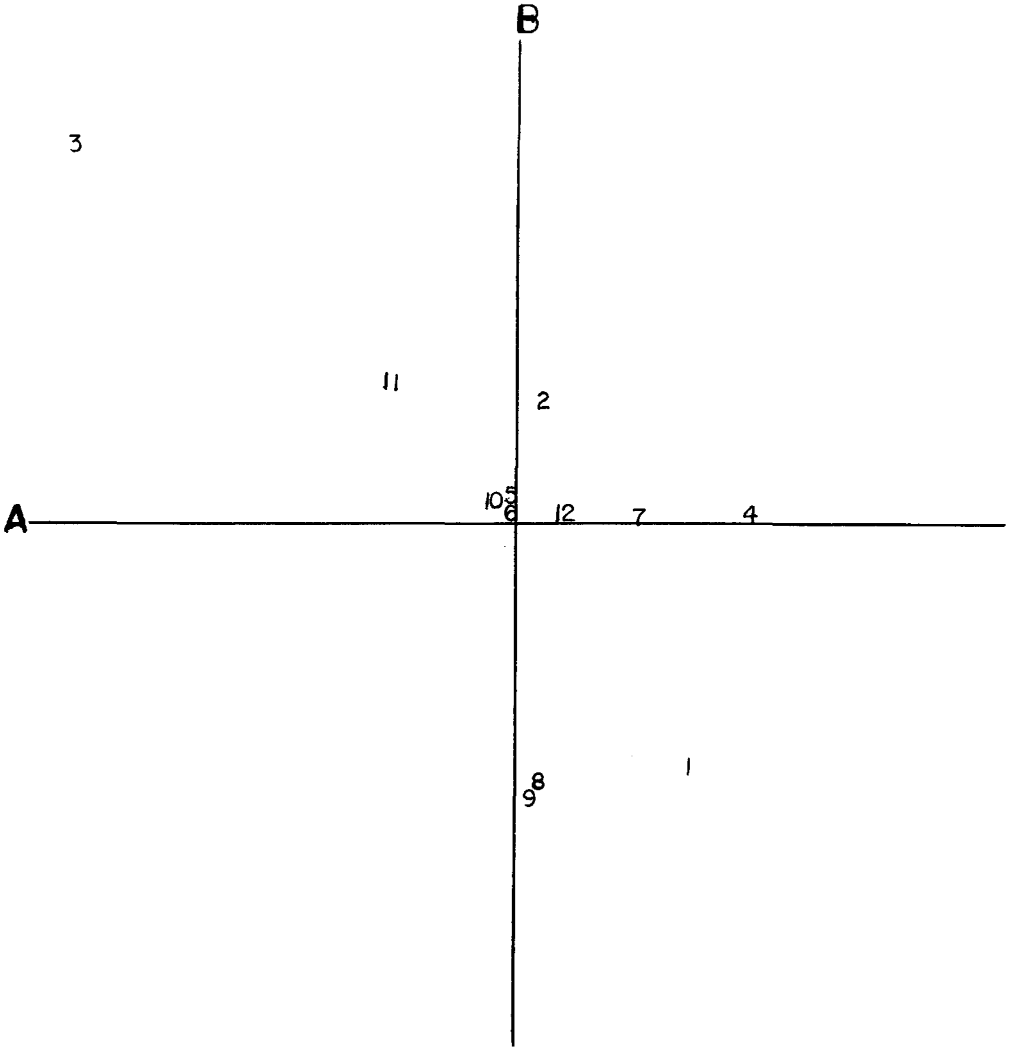


Fig. 10. Plots of loadings on final rotated factors.

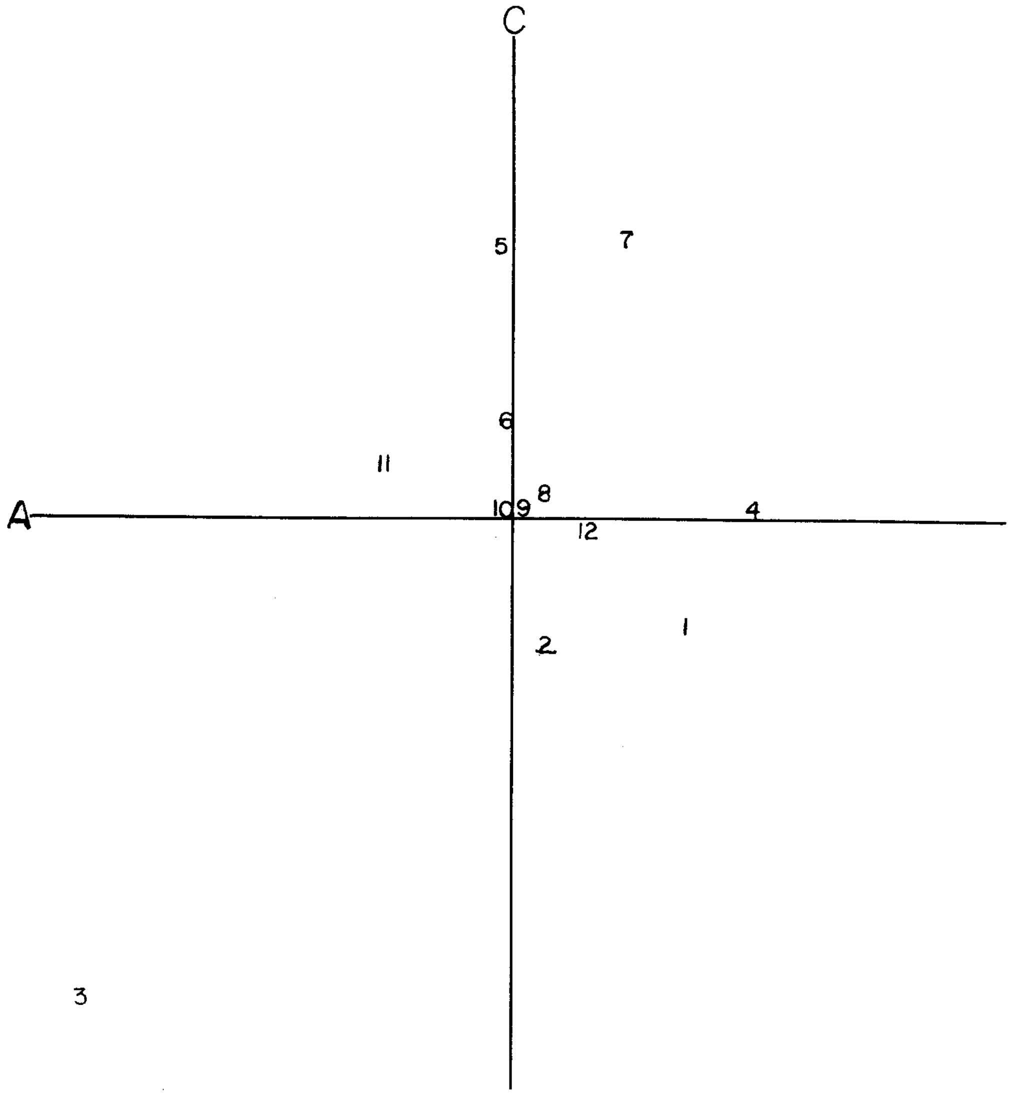
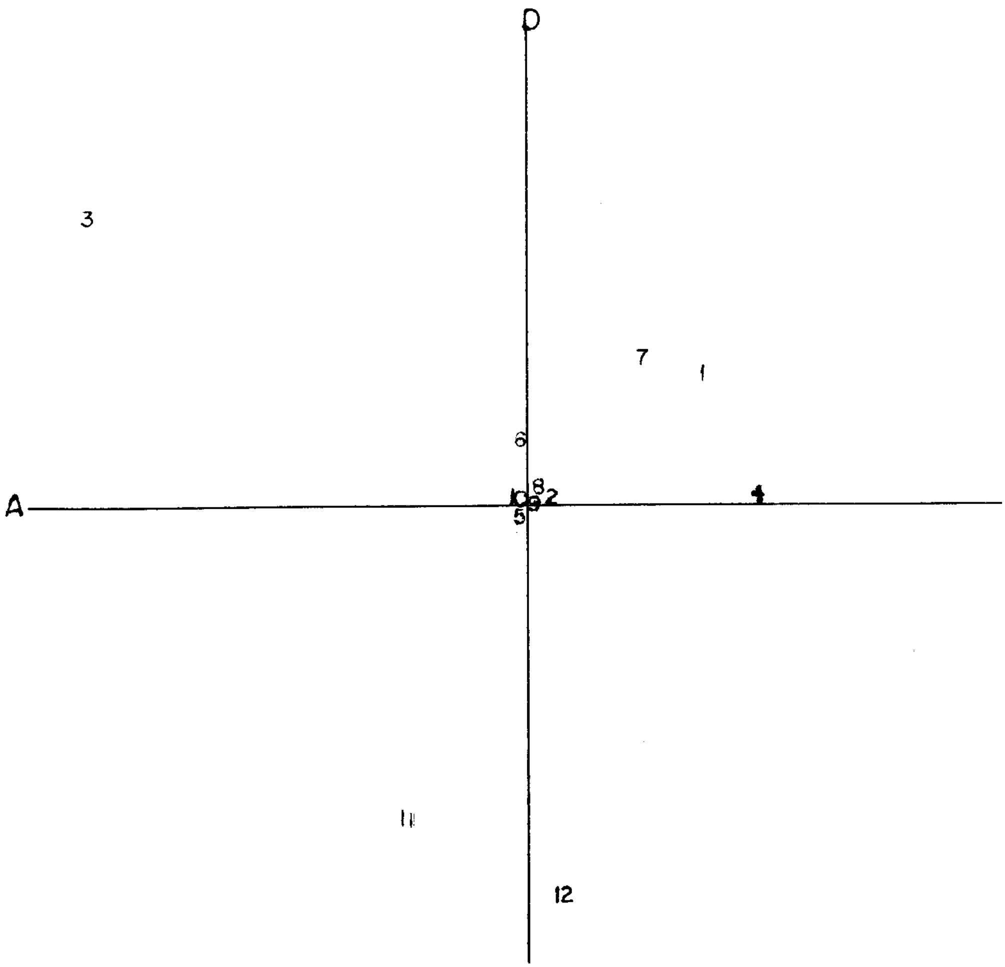


Fig. 10. Continued.



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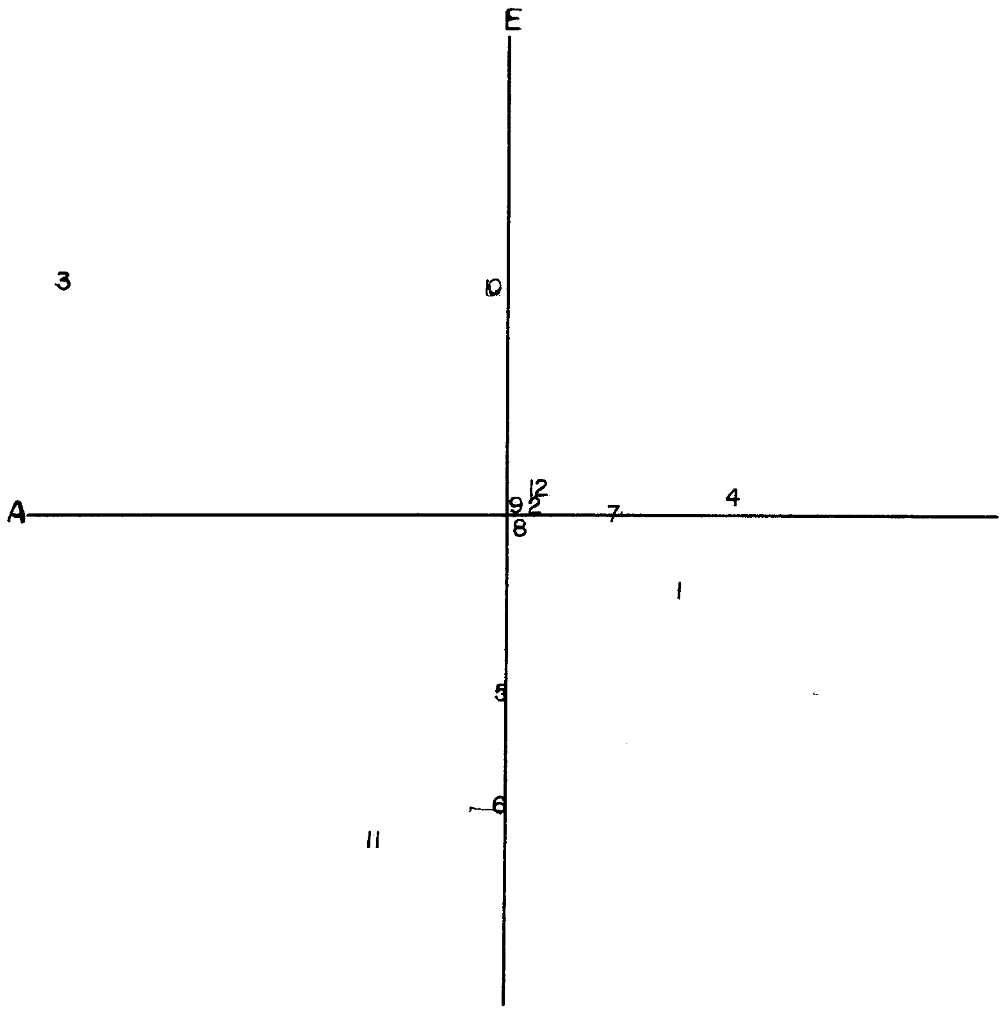

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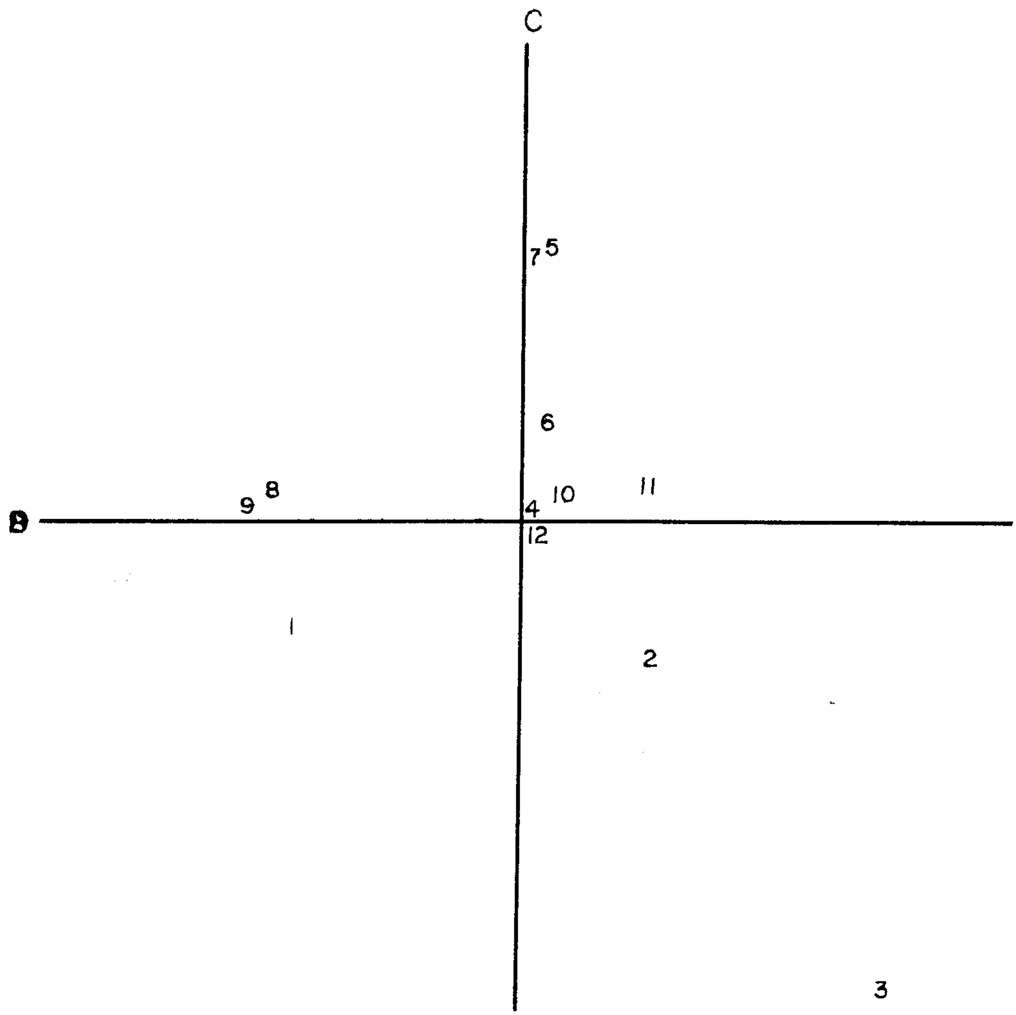

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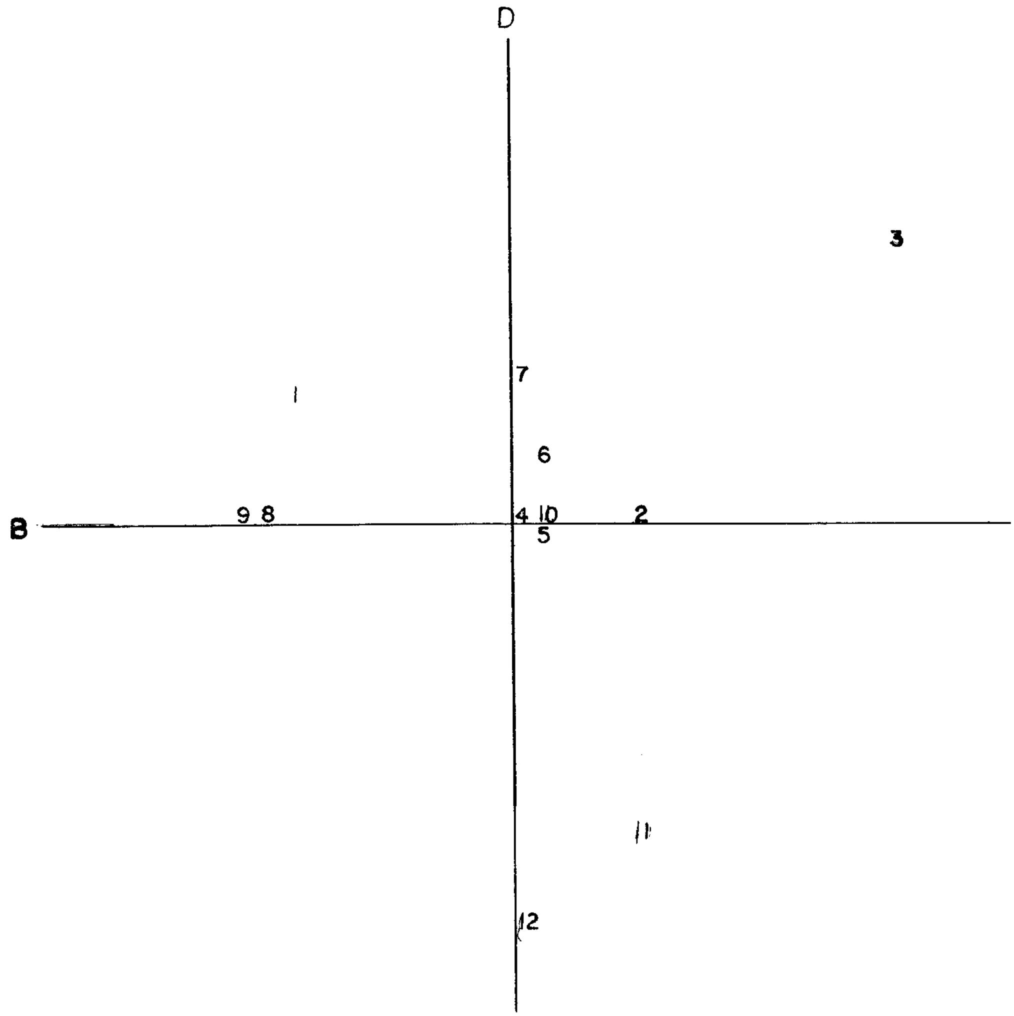

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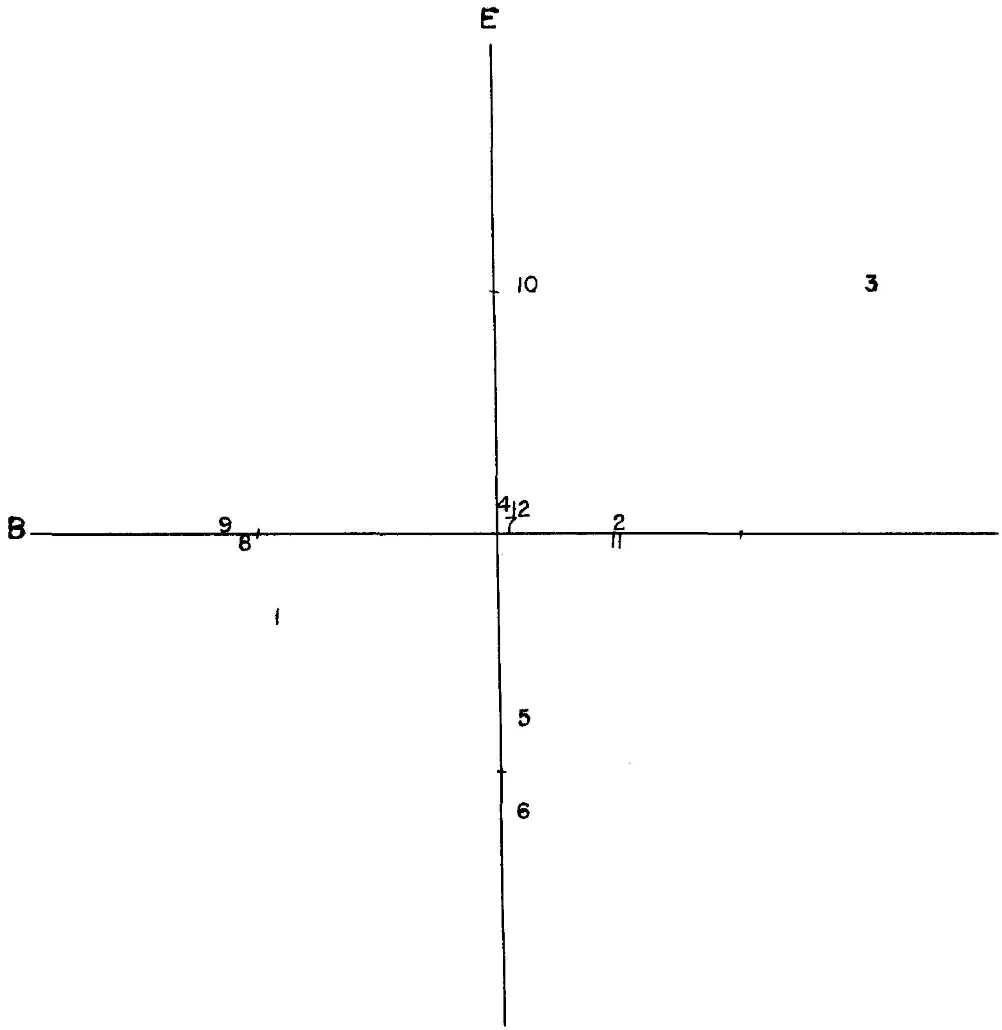

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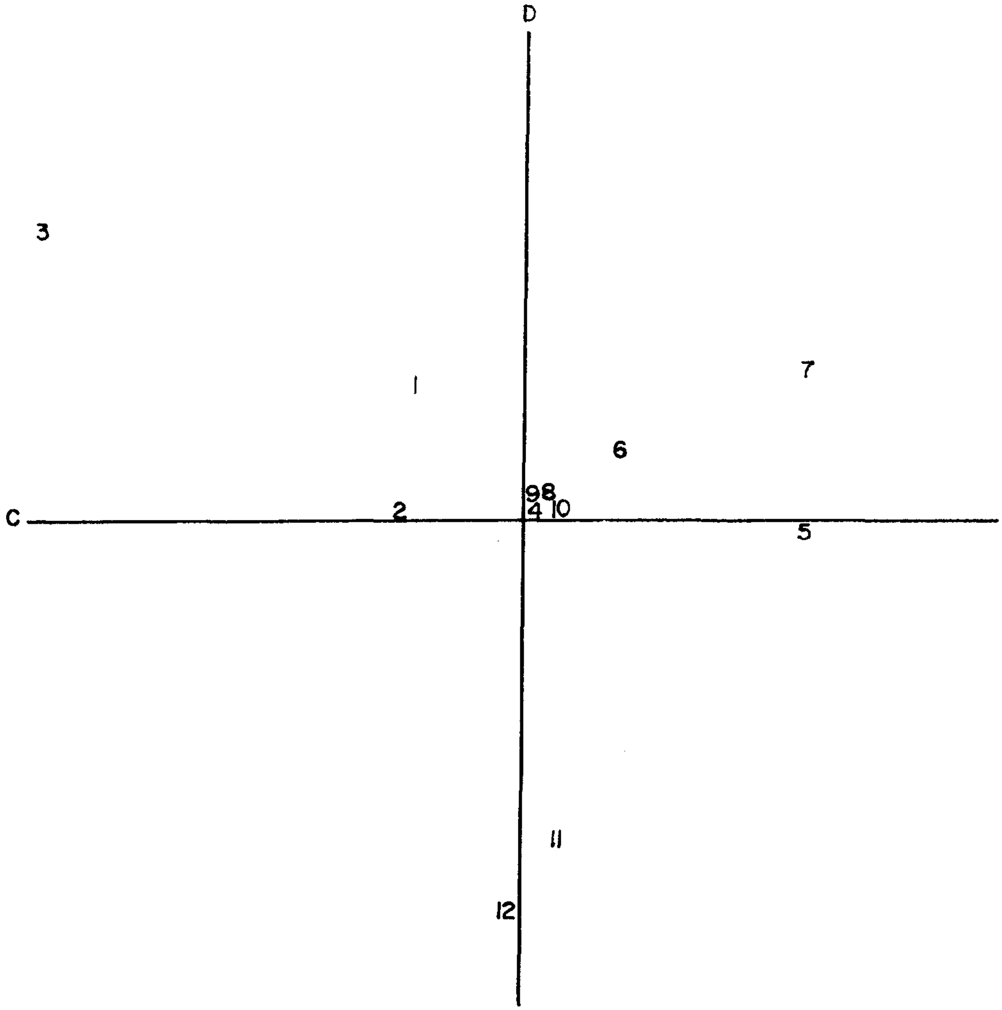

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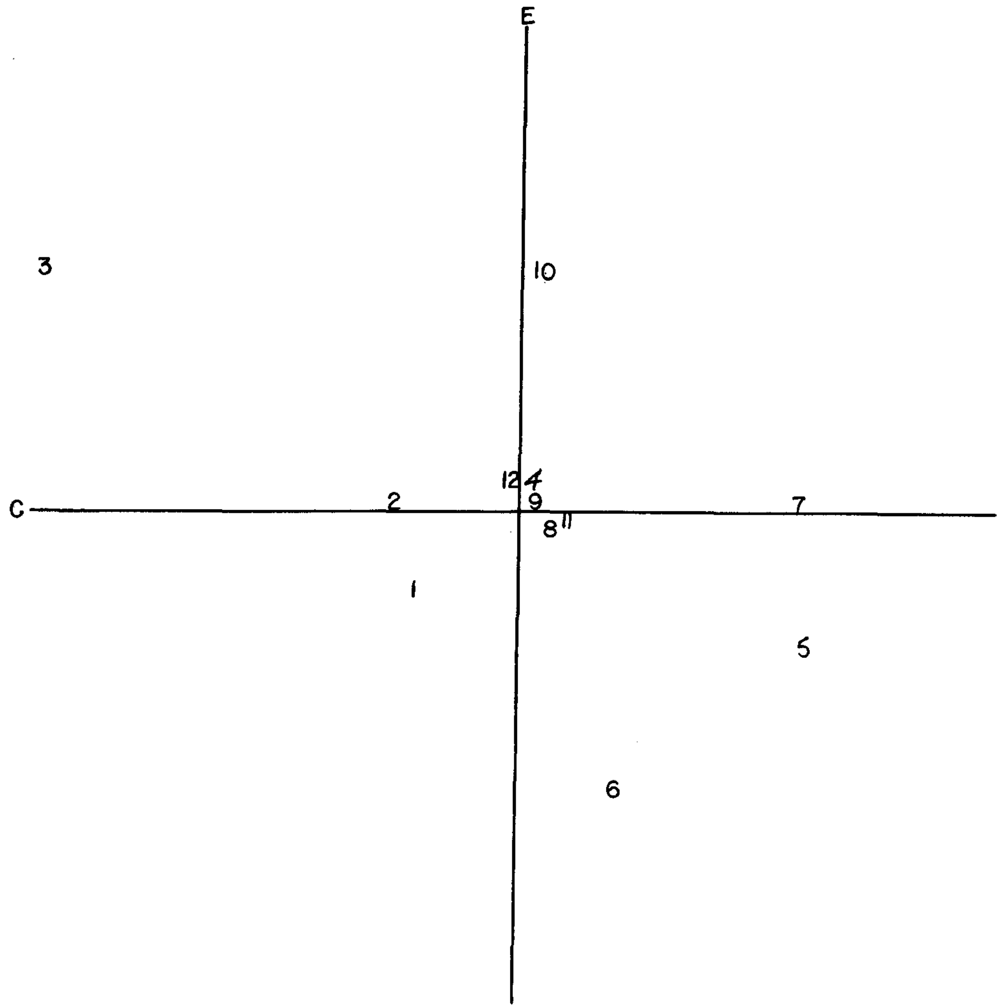

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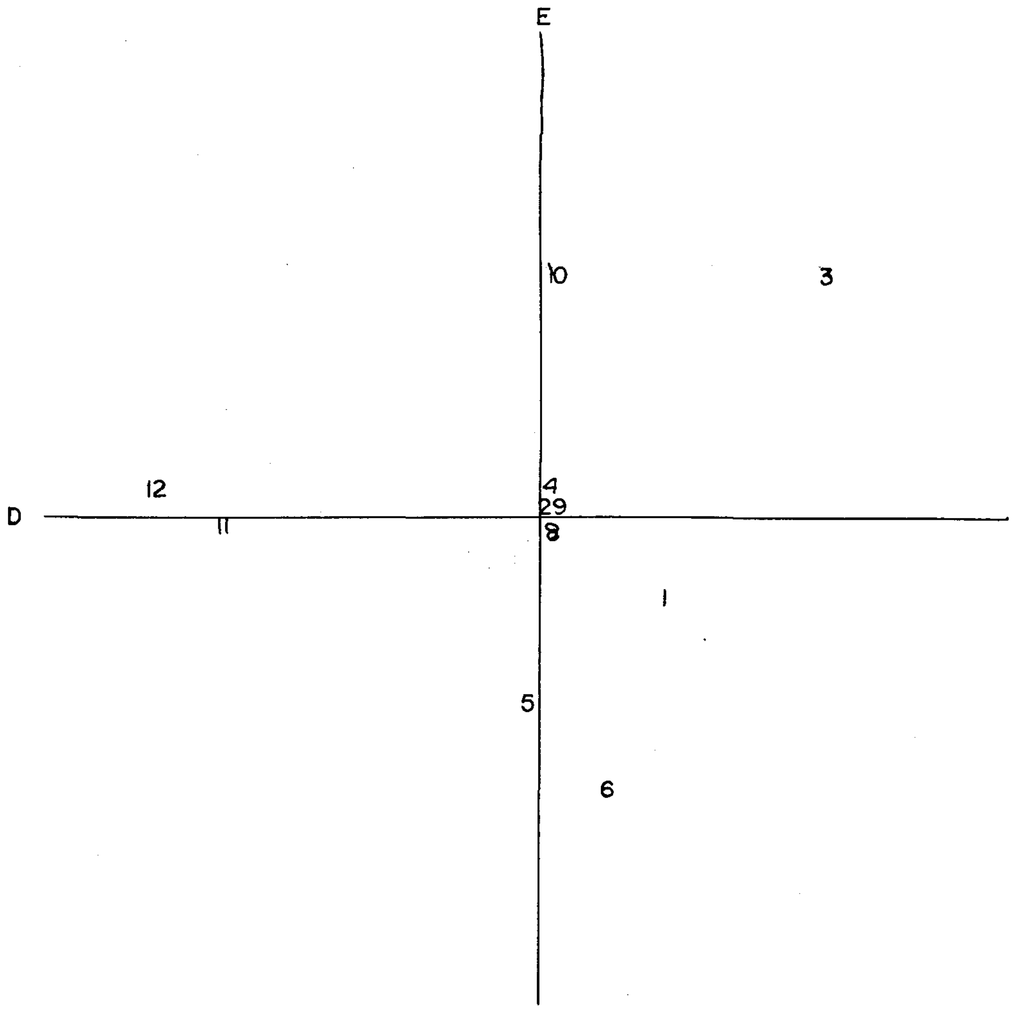

Fig. 10. Continued.

APPROVAL SHEET

The thesis submitted by Isabel O. Reyes has been read and approved by three members of the Department of Psychology.

The final copies have been examined by the director of the thesis and the signature which appears below verifies the fact that any necessary changes have been incorporated, and that the thesis is now given final approval with reference to content, form, and mechanical accuracy.

The thesis is therefore accepted in partial fulfillment of the requirements for the Degree of Master of Arts.

Date

Signature of Adviser